Rayat Shikshan Sanstha’s

Karmaveer Bhaurao Patil College Vashi, Navi Mumbai

Autonomous College

[University of Mumbai]

Syllabus for Approval

Sr. No.	Heading	Particulars
1	**Title of Course**	**M.Sc. II Mathematics**
2	**Eligibility for Admission**	**M.Sc. I**
3	**Passing Marks**	**40%**
4	**Ordinances/Regulations (if any)**	
5	**No. of Years/Semesters**	**One year/Two semesters**
6	**Level**	**P.G.**
7	**Pattern**	**Semester**
8	**Status**	**Revised**
9	**To be implemented from Academic year**	**2020-2021**

AC- //

Item No-

Rayat Shikshan Sanstha's

KARMAVEER BHAURAO PATIL COLLEGE, VASHI.

NAVI MUMBAI

(**AUTONOMOUS COLLEGE**)

Sector-15- A, Vashi, Navi Mumbai - 400 703

Syllabus for M.Sc. II Mathematics

Program: M.Sc.

Course: M.Sc. II Mathematics

(Choice Based Credit, Grading and Semester System with effect from the academic year 2020-2021)

Preamble of the Syllabus:

Master of Science (M.Sc.) in Mathematics is a post graduation programme of Department of Mathematics, Karmaveer Bhaurao Patil College Vashi, Navi Mumbai [Autonomous College]

The Choice Based Credit and Grading System to be implemented through this curriculum, would allow students to develop a strong footing in the fundamentals and specialize in the disciplines of his/her liking and abilities. The students pursuing this course would have to develop understanding of various aspects of the mathematics. The conceptual understanding, development of experimental skills, developing the aptitude for academic and professional skills, acquiring basic concepts and understanding of hyphenated techniques are among such important aspects.

Rayat Shikshan Sanstha's
KARMAVEER BHAURAO PATIL COLLEGE, VASHI
[AUTONOMOUS COLLEGE]

Department of Mathematics

M.Sc. Mathematics
Choice Based Credit System (CBCS)

SEMESTER – III

Course Code	Unit	Topic	Credit	L/W
		Algebra III		
PGMT301	I	Groups	6	4
	II	Representation of finite groups		
	III	Modules		
	IV	Modules over PID		
		Analysis II		
PGMT302	I	Measures	6	4
	II	Measurable functions		
	III	Integration of measurable functions		
	IV	Convergence theorem and L^P (μ) spaces		
		Optimization Techniques		
PGMT303	I	Linear Programming	6	4
	II	Unconstrained Optimization I		
	III	Unconstrained Optimization II		
	IV	Constrained Optimization Problems		
PGMT304		**ELECTIVE COURSE I**	3	4
PGMT305		**ELECTIVE COURSE II**	3	4

SEMESTER IV

Course Code	Unit	Topic	Credit	L/W
Field Theory				
PGMT401	I	Algebraic Extensions	6	4
	II	Normal and Separable Extensions		
	III	Galois Theorems		
	IV	Applications		
Functional Analysis				
PGMT402	I	Baire spaces, Hilbert spaces	5	4
	II	Normed Linear Spaces		
	III	Bounded Linear Transformations		
	IV	Basic Theorems		
Partial Differential Equations				
PGMT403	I	Classification of second order Linear partial differential equations	5	4
	II	Laplace operator		
	III	Heat operator		
	IV	Wave operator		
PGMT404		**ELECTIVE COURSE I**	4	4
PGMT405		**ELECTIVE COURSE II**	4	4
PGMT406		**PROJECT COURSE**	4	4

Teaching Pattern for Semester III and IV:

1. Four lectures per week per course. Each lecture is of 60 minutes duration.
2. In addition, there shall be tutorials, seminars as necessary for each of the five courses.
3. PGMT406 is a project based Course for Semester IV. The projects for this course are to be guided by the Faculty members. Each project shall have maximum of 08 (eight) students. The workload for each project is 1L/W.

SEMESTER III

All Results have to be done with proof unless otherwise stated.

PGMT301: Algebra III

Learning Outcomes:

1. Understand the concept of modules with examples and their role in mathematics.
2. Prove the basic results of module theory.
3. Identify and analyze different types of algebraic structures such as solvable groups, simple groups and alternate groups to understand and use the fundamental results in algebra.
4. State Maschke's theorem and Schur's lemma.
5. Define character of a linear representation and list the properties exhibited by them.
6. Find the character table of groups of small order.
7. Define submodules and annihilator ideals.
8. Explain the concept of module homomorphism, kernels and quotient modules and state the isomorphism theorems.
9. Explain the concept of free modules and the matrix representation of homomorphism between free modules of finite ranks.
10. Learn theory of modules over PID and its applications to Jordan and Rational canonical forms.
11. Explain the structure theorem for finitely generated modules over a ring and its applications to abelian groups and matrices.

Unit I. Groups (15 Lectures)

Simple groups, A_5 is simple. Solvable groups, Solvability of all groups of order less than 60, Nilpotent groups, Isomorphism theorem, Jordan-Holder theorem, Direct and Semi-direct products, Examples such as group of affine transformations and Dihedral groups as semi-direct product. Classification of groups of finite order (upto 21).

Unit II. Representation of finite groups (15 Lectures)

Linear representations of a finite group on a finite dimensional vector space over$C$, Maschke's theorem. The space of class functions, Characters and Orthogonality relations. Irreducible representations of finite groups and Schur's lemma.

Character tables with emphasis on examples of groups of small order.

Reference: Chapter 2, 3 from Benjamin Steinberg, Representation Theory of Finite groups.

Unit III. Modules (15 Lectures)

Modules over rings, Abelian groups as Z -modules, Submodules. Annihilators. Module homomorphisms, kernels. Quotient modules. Isomorphism theorems. Generating sets of modules and finitely generated modules, (internal) direct sums and equivalent conditions. Free modules, free module of rank n. Invariant basis property of commutative ring.

Matrix representations of homomorphisms between free modules of finite ranks.

Unit IV. Modules over PID (15 Lectures)

Noetherian modules and equivalent conditions. Torsion elements of a module, torsion free modules, submodule of a free module over a PID, Structure theorem for finitely generated modules over a PID: Fundamental theorem, Existence (Invariant Factor Form and Elementary Divisor Form), Fundamental theorem, Uniqueness. Applications to the Structure theorem for finitely generated Abelian groups and linear operators.

Reference Books:

1. D.S. Dummit and R.M. Foote, Abstract Algebra, John Wiley and Sons.
2. S. Lang, Algebra, Springer Verlag, 2004
3. N. Jacobson, Basic Algebra, Volume 1, Dover, 1985.
4. M. Artin, Algebra, Prentice Hall of India.

5. Benjamin Steinberg, Representation Theory of Finite groups.

PGMT302 ANALYSIS II

Learning Outcomes:

1. Understand how Lebesgue measure on R^d is defined.
2. Understand how measures may be used to construct integrals.
3. Explain results on outer measure in R^d.
4. Understand basic properties of measurable functions.
5. Establish measurability or non-measurability of sets and functions.
6. Explain the properties of measurable functions.
7. Have knowledge of Lebesgue integration, convergence theorems for Lebesgue integrals and Fubini's theorem.
8. Compute Lebesgue integral and have knowledge of its applications to volume calculations and Fourier analysis.
9. Know the basic convergence theorems for the Lebesgue integral.
10. Understanding that Lebesgue integration can solve certain problems for which Riemann integration does not provide adequate answers.

Unit I: Measures (15 Lectures)

Outer measure μ^* on X, μ^* measurable subsets on X (A subset E of a set X with outer measure μ^* is said to be μ^* measurable if $\mu^*(A) = \mu^*(A \cap E) + \mu^*\big(A \cap (X \backslash E)\big) \forall A \subseteq X$), the collection $\sum$ of all μ^*-measurable subsets of X form a σ-algebra, measure space $\left(X, \sum, \mu\right)$.
Volume $\lambda(I)$ of any rectangle in R^d(for interval $I = \prod_{i=1}^{d} (a_i, b_i)$of R^d, $\lambda(I) = \prod_{i=1}^{d} (b_i - a_i)$.
Lebesgue's Outer measure m^*in $R^d$and results:

1. Lebesgue's outer measure m^* is translation invariant.
2. Let A, B be two subsets of R^d with $d(A, B) > 0$. Then $m^*(A \cup B) = m^*(A) + m^*(B)$.
3. For any bounded interval $I = (a, b)$ of $R, m^*(I) = b - a$.
4. For any interval $I\ of\ R^d, m^*(I) = \lambda(I)$.

The σ −algebra $M$of all Lebesgue measurable subsets of R^d, the lebesgue measure $m = m^*|_M$ and the measure space (R^d, M, m).
Borel $\sigma - algebra$ of R^d, Any closed subset and any open subset of $R^d$is Lebesgue measurable. Every Borel set in R^d is Lebesgue measurable. For any bounded Lebesgue measurable subset E of R^d , given $\epsilon > 0$ there exist compact set K and open set U in R^d, such that $K \subseteq E \subseteq U\ and\ m(U - K) < \in$. for any Lebesgue measurable subset E of R^d there exist Borel set F, G in R^d such that, $F \subseteq E \subseteq G\ and\ m(E - F) = 0 = m(G - E)$.
Existence of subset of R which is not Lebesgue measurable.F^σ sets, G^δ sets.

Unit II. Measurable functions (15 lectures)

Measurable function on $(X, \sum, \mu)$, simple functions, properties of measurable functions. If $f \geq 0$ is a measurable function, then there exist a monotone increasing sequence (S_n) of a non-negative simple measurable functions converging to pointwise to the function f.Convergence in measure.
Complex valued Lebesgue measurable functions in R^d, Lusin's Theorem(Every measurable function is almost a continuous function.)

Unit III. Integration of measurable functions

Integral $\int_X S d\mu$of a non-negative simple measurable function S defined on the measure space (X,∑,μ) and properties, integral of non-negative measurable function, Monotone convergence theorem. If $f \geq 0$ and $g \geq 0$ are measurable functions, then $\int_X (f + g) d\mu = \int_X f\, d\mu + \int_X g\, d\mu$. Lebesgue integral of complex valued measurable functions, approximation of Lebesgue integrable functions by continuous functions with compact support.

Unit IV. Convergence theorem and L^P (μ)spaces (15 lectures)

Monotone convergence theorem, Fatou's lemma, Dominated convergence theorem, Completeness of $L^1(\mu), L^2(\mu)$

Lebesgue's and Riemann integrals: A bounded real valued function on $[a, b]$ is Riemann integrable if and only if it is continuous at almost every point of $[a, b]$ in this case, its Riemann integral and Lebesgue integral coincide. Product measures and Fubini's theorem.

Recommended Textbooks:

1. H.L.Royden, Real Analysis by PHI
2. Andrew Browder, Mathematical Analysis, An Introduction, Springer Undergraduate Texts in Mathematics
3. Walter Rudin, Real and Complex Analysis, McGraw Hill India, 1974.
4. E. Stein & R. Shakarchi, Real Analysis.

PGMT303: Optimisation

Learning Outcomes:

1. Apply the knowledge of basic optimization techniques in order to get best possible results from a set of several possible solutions of different problems viz. linear programming problems, transportation problems, assignment problem and unconstrained and constrained problem.
2. Formulate an optimization problem from its physical conditions.
3. Select and implement an appropriate optimization technique keeping in mind its limitations in order to solve a particular optimization problem.
4. Understand theoretical foundation and implementation of similar type optimization techniques available in scientific literature.
5. Continue to acquire knowledge and skills of optimization techniques that are appropriate to professional activities.
6. Extend their knowledge of basic optimization techniques to do interesting research work on these types of optimization techniques.
7. Explain the theory of simplex method, Dual simplex method and solve problems.
8. Explain the various search methods such as one-dimensional search method, Fibonacci search, Newton's method and Secant method and solve problems.
9. Explain various methods of unconstrained optimization such as Powell's method, Nelder-Mead method etc. and solve problems using appropriate methods.
10. Explain the Lagrange multiplier theorem, Karush-Kuhn-Tucker theorem.
11. State the second order necessary conditions for equality and inequality constraint problems.

Unit I. Linear Programming(15 Lectures)

Operations research and its scope, Necessity of operations research in industry, Linear programming problems, Convex sets, Simplex method, Theory of simplex method, Duality theory and sensitivity analysis, Dual simplex method.

Unit II. Unconstrained Optimization I (15 Lectures)

First and second order conditions for local optima, One-Dimensional Search Methods: Golden Section Search, Fibonacci Search, Newton's Method, Secant Method

Unit III. Unconstrained Optimization II (15 Lectures)

Powell's Method, Nelder-Mead (Simplex Method),Gradient Methods: Steepest Descent Methods, Newton's method, Conjugate gradient methods.

Unit IV. Constrained Optimization Problems(15 Lectures)

Problems with equality constraints, Tangent and normal spaces, Lagrange Multiplier Theorem, Second order conditions for equality constraints problems, Problems with inequality constraints, Karush-Kuhn-Tucker Theorem, Second order necessary conditions for inequality constraint problems.

Recommended Text Books:

1.H.A. Taha, Operations Research-An introduction, Macmillan Publishing Co. Inc., NY.
2.K. Swarup, P. K. Gupta and Man Mohan, Operations Research, S. Chand and sons, New Delhi.
3.S.S. Rao, Optimization Theory and Applications, Wiley Eastern Ltd, NewDelhi.
4.G. Hadley, Linear Programming, Narosa Publishing House, 1995.
5.F.S. Hillier and G.J. Lieberman, Introduction to Operations Research (Sixth Edition), McGraw Hill.

PGMT304& PGMT305: ELECTIVE COURSES

Elective Courses to be selected from the following four courses.

1. Numerical Methods I

All methods discussed in the course should be illustrated using suitable programming language and conducting practicals.

Learning Outcomes:

1. Analyze the errors obtained in the numerical solution of problems.
2. Represent numbers in binary, decimal, octal and hexadecimal forms.
3. Define relative, absolute and percentage errors. Find errors in different iterative methods.
4. Apply iterative methods based on first degree equation such as Newton Raphson method, secant method etc., to find roots of polynomial.
5. Find rate of convergence of various iterative methods.
6. Analyze the errors obtained in the numerical solution of problems.
7. Using appropriate numerical methods, determine approximate solutions to systems of linear equations.
8. Express the given system of linear equation in matrix form and apply Gaussian method to find the solution of the given system.
9. Use Triangularization methods such as Doolittle and Crout's method, Cholesky method etc., to find the solution of system of linear equations.
10. Find the largest and smallest Eigen value of a matrix using power method.
11. Find the difference of polynomial.
12. Perform interpolation such as linear, quadratic and cubic interpolation to find the polynomial.
13. Derive Newton forward formula and Newton backward formula.
14. Solve problems using Newton forward formula and Newton backward formula.

Unit I. Basics of Numerical Analysis (15 Lectures)
Error in numerical computations, Absolute, Relative and percentage errors, Round off errors, Truncation errors, Inherent errors, Representation of numbers: Binary, Octal, Decimal, Hexadecimal.

Unit II. Solution of Algebraic & Transcendental Equations (15 Lectures)
Iteration method, Newton-Raphson method, Muller's method, Ramanujan's method, Chebyshev method, Rate of convergence, Solution of polynomial equations, Solutions of nonlinear equations: Seidel iteration, Newton-Raphson method.

Unit III. System of linear equations and solutions (15 Lectures)
Gaussian elimination, Gauss-Jordan method, Triangularization method: Crout's method, Cholesky method, Iteration methods: Gauss-Jacobi, Gauss-Seidel, Eigen value problem for matrices: Power method, Inverse power method, Jacobi or Given's method for real symmetric matrices, Singular value decomposition.

Unit IV. Interpolation (15 Lectures)

Difference operators, Lagrange's interpolation formula, Divided difference formula, Newton's forward and backward difference interpolation formulae, Error in interpolating polynomial, Spline interpolation, Numerical differentiation, Maxima and minima of interpolating polynomial.

Reference Books

(1) H.M.Antia, Numerical Analysis for Scientists and Engineers, TMH 1991.
(2) Jain, Iyengar, Numerical methods for Scientific and Engineering problems, New Age Inter- national, 2007.
(3) S.S.Sastry, Introductory methods of Numerical Analysis, Prentice-Hall India, 1977.
(4) K.E. Atkinson, An introduction to Numerical Analysis, John Wiley and sons, 1978.

2. Graph Theory I

Learning Outcomes:

1. Describe the origin of Graph Theory.
2. Define general graph, Directed and Undirected graph, Simple and multiple graph.
3. Compare and contrast different types of graphs such as complete graphs, Null graph, Complementary graphs, Regular graphs, etc.
4. Explain the concept of isomorphism in graph theory and its consequences.
5. Solve problems involving vertex and edge connectivity.
6. Characterize tree.
7. Determine the cut edge and cut vertices of a graph.
8. Use BFS and DFS algorithm to find the shortest path.
9. Use Kruskal's algorithm to find minimal spanning tree.
10. Derive some properties of planarity and Euler's formula.
11. Characterize Eulerian and Hamiltonian graphs.
12. Find Hamiltonian closure of a graph.
13. State Dirac's theorem and Chvatal theorem.
14. Define augmenting path, matchings and matchings in a bipartite graph.
15. Find Ramsey number.
16. Understand Tutte's theorem, Berge theorem and Ramsey theorem.

Unit I. Connectivity(15 Lectures)

Overview of Graph theory-Definition of basic concepts such as Graph, Subgraphs, Adjacency and incidence matrix, Degree, Connected graph, Components, Isomorphism, Bipartite graphs etc., Shortest path problem-Dijkstra's algorithm, Vertex and Edge connectivity-Result $k \leq k' \leq \delta$, Blocks, Block-cut point theorem, Construction of reliable communication network, Menger's theorem.

Unit II. Trees (15 Lectures)

Trees-Cut vertices, Cut edges, Bond, Characterizations of Trees, Spanning trees, Fundamentalcycles, Vector space associated with graph, Cayley's formula, Connector problem- Kruskal'salgorithm, Proof of correctness, Binary and rooted trees, Huffman coding, Searching algorithms-BFS and DFS algorithms.

Unit III. Eulerian and Hamiltonian Graphs (15 Lectures)

Eulerian Graphs- Characterization of Eulerian Graph, Randomly Eulerian graphs, Chinese postman problem- Fleury's algorithm with proof of correctness. Hamiltonian graphs- Necessary condition, Dirac's theorem, Hamiltonian closure of a graph, Chvatal theorem, Degree majorisation, Maximum edges in a non-hamiltonian graph, Traveling salesman problem.

Unit IV. Matching and Ramsey Theory (15 Lectures)

Matchings-augmenting path, Berge theorem, Matching in bipartite graph, Halls theorem, Konig's theorem, Tutte's theorem, Personal assignment problem, Independent sets and covering- $\alpha + \beta = p$; Gallai's theorem, Ramsey theorem-Existence of r(k, l); Upper bounds of r(k, l); Lower bound for mr(k, l) $\geq 2^{m/2}$ where m = min{k, l}; Generalize Ramsey numbers-r(k_1, k_2,…, k_n); Graph Ramsey theorem, Evaluation of r(G; H) when for simple graphs G = P_3, H = C_4

Recommended Text Books:

1. J.A. Bondy and U.S.R. Murty, Graph Theory with Applications, Elsevier.
2. J. A. Bondy and U.S. R. Murty, Graph Theory, GTM 244 Springer, 2008.
3. M. Behzad and A. Chartrand, Introduction to the Theory of Graphs, Allyn and Becon Inc., Boston, 1971.
4. K. Rosen, Discrete Mathematics and its Applications, Tata-McGraw Hill, 2011.
5. D.B.West, Introduction to Graph Theory, PHI, 2009.

C. Design Theory

Learning Outcomes:
1. Explain the concept of design theory and state its properties.
2. Find incidence matrix.
3. Understand the concept of isomorphism and automorphism.
4. Construct BIBDs.
5. Understand the intersection property and the Bruck-Ryser-Chowla theorem.
6. Find quadratic residues.
7. Explain the multiplier theorem with proof.
8. Define Hadamard matrices and explain the equivalence between Hadamard matrices and BIBDs.
9. Understand the results for Hadamard matrices for small orders.

Unit I. Introduction to Balanced Incomplete Block Designs (15 Lectures)

What Is Design Theory? Basic Definitions and Properties, Incidence Matrices, Isomorphisms and Automorphisms, Constructing BIBDs with Specified Automorphisms, New BIBDs from Old, Fishers Inequality.

Unit II. Symmetric BIBDs (15 Lectures)

An Intersection Property, Residual and Derived BIBDs, Projective Planes and Geometries, The Bruck-Ryser-Chowla Theorem. Finite affine and projective planes.

Unit III. Difference Sets and Automorphisms (15 Lectures)

Difference Sets and Automorphisms, Quadratic Residue Difference Sets, Singer Difference Sets, The Multiplier Theorem, Multipliers of Difference Sets, The Group Ring, Proof of the Multiplier Theorem, Difference Families, A Construction for Difference Families.

Unit IV. Hadamard Matrices and Designs (15 Lectures)

Hadamard Matrices, An Equivalence Between Hadamard Matrices and BIBDs, Conference Matrices and Hadamard Matrices, A Product Construction, Williamson's Method, Existence Resultsfor Hadamard Matrices of Small Orders, Regular Hadamard Matrices, Excess of Hadamard Matrices, Bent Functions.

Recommended Text Books:

1. D. R. Stinson, Combinatorial Designs: Constructions and Analysis, Springer,2004.
2. W.D. Wallis, Introduction to Combinatorial Designs, (2nd Ed), Chapman & Hall.
3. D. R. Hughes and F. C. Piper, Design Theory, Cambridge University Press, Cam-bridge, 1985.

4. T. Beth, D. Jung nickel and H. Lenz, Design Theory, Volume 1 (Second Edition), Cambridge University Press, Cambridge, 1999.

D. Integral Transforms

Learning Outcomes:
1. Understand the theory and applications of integral transforms.
2. Explain how integral transforms can be used to solve a variety of differential equations.
3. Develop their attitude towards problem solving.
4. Define Laplace transform, find Laplace transform of some elementary functions and state properties of Laplace transform.
5. Define Fourier and inverse Fourier transforms and state properties of Fourier transforms.
6. Understand the Fourier integral theorem and convolution theorem.
7. Apply Fourier transforms to find the solution of initial and boundary value problems.
8. State the properties of Mellin transform.
9. Understand the convolution theorem for Mellin transform.
10. Define Z-transform, inverse of Z-transform.
11. Find solutions of difference equations using Z-transform.

Unit I. Laplace Transform (15 Lectures)
Definition of Laplace Transform, Laplace transforms of some elementary functions, Properties of Laplace transform, Laplace transform of the derivative of a function, Inverse Laplace Transform, Properties of Inverse Laplace Transform, Inverse Laplace Transform of derivatives, Convolution Theorem, Heaviside's expansion theorem, Application of Laplace transform to solutions of ODEs and PDEs.

Unit II. Fourier Transform (15 Lectures)
Fourier Integral theorem, Properties of Fourier Transform, Inverse Fourier Transform, Convolution Theorem, Fourier Transform of the derivatives of functions, Parseval's Identity, Relationship of Fourier and Laplace Transform, Application of Fourier transforms to the solution of initial and boundary value problems.

Unit III. Mellin Transform (15 Lectures)
Properties and evaluation of Mellin transforms, Convolution theorem for Mellin transform, Complex variable method and applications.

Unit IV. Z-Transform (15 Lectures)
Definition of Z-transform, Inversion of the Z-transform, Solutions of difference equations using Z-transform. Applications.

Recommended Text Books:

1. Brian Davies, Integral transforms and their Applications, Springer.
2. L. Andrews and B. Shivamogg, Integral Transforms for Engineers, Prentice Hall of India.
3. I.N.Sneddon, Use of Integral Transforms, Tata-McGraw Hill.
4. R. Bracemell, Fourier Transform and its Applications, MacDraw hill.

SEMESTER IV

PGMT401: Field Theory

Learning Outcomes:
1. Understand the notion of extension of field.

2. Define an algebraic element and algebraic extension. Deduce some basic results such as every finite extension is algebraic.
3. Define straightedge and compass construction and explain few results.
4. Explain the impossibility of classical Greek problems.
5. Understand the properties of finite fields.
6. Find splitting fields of various polynomials and observe that the splitting field of a polynomial is unique up to isomorphism.
7. Explain the concept of separable extension.
8. Create, select and apply appropriate algebraic structures such as Galois extension, Automorphisms of groups of fixed fields, Fundamental theorem of Galois Theory to understand and use the Fundamental theorem of Algebra.
9. Find the Galois group corresponding to a Galois extension.
10. Define radical extension and give examples of a radical extension.
11. Observe that every radical extension is an n radical extension for some n.
12. Solve examples on solvability by radicals.

Unit I. Algebraic Extensions (15 lectures)

Prime sub field of a field, definition of field extension K/F, algebraic elements, set of algebraic elements in a field extension, $F[\alpha]$ as a quotient of $F[x]$, minimal polynomial of an algebraic element, extension of a field obtained by adjoining one algebraic element. Algebraic extensions, Finite extensions, degree of an algebraic element, degree of a field extension. Tower law for field extensions, Transitivity of algebraic extensions. Composite field of two sub fields of a field and examples. (Ref: D.S. Dummit and R.M. Foote, Abstract Algebra).

Unit II. Normal and Separable Extensions (15 lectures)

Splitting field for a set of polynomials, normal extension, examples such of splitting fields of $x^p - 2$ (p prime), uniqueness of splitting fields, existence and uniqueness of finite fields. Algebraic closure of a field, existence of algebraic closure.

Separable elements, Separable extensions, example of non-separable extension. Frobenius automorphism of a finite field. Separability of finite fields, Primitive element theorem.

Unit III. Galois Theory (15 Lectures)

Galois group G(K/F) of a field extension K/F, Galois extensions, Subgroups, Fixed fields, Galois correspondence, Fundamental theorem of Galois theory.

Unit IV. Applications (15 Lectures)

Classical Straight-edge and Compass constructions: definition of Constructible points, lines, circles by Straight-edge and Compass starting with (0, 0) and (1, 0); definition of constructible real numbers. If $a \in R$ is constructible, then α is an algebraic number and its degree over Q is a power of 2. Cos(20°) is not a constructible number. The regular 7-gon is not constructible.

The regular 17-gon is constructible. The Constructible numbers form a sub field of R. If a > 0 is constructible, then so is $\sqrt{a}$. Impossibility of the classical Greek problems: 1) Doubling a Cube, 2) Trisecting an Angle, 3) Squaring the Circle is possible.

Cyclotomic field $Q(\zeta_n)$ (splitting eld of x^n-1 over Q), cyclotomic polynomial, degree of Cyclotomic field $Q(\zeta_n)$ D.S. Dummit and R.M. Foote, Abstract Algeba). Galois group for an irreducible cubic polynomial, Galois group for an irreducible quadratic polynomial. (Ref: M. Artin, Algebra, Prentice Hall of India).

Solvability by radicals in terms of Galois group and Abel'stheorem on the insolvability of a general quintic.

Rcommended Text Books:

1. D.S. Dummit and R.M. Foote, Abstract Algebra, John Wiley and Sons.
2. M. Artin, Algebra, Prentice Hall of India, 2011.

Additional Reference Books:

1. S. Lang, Algebra, Springer Verlag, 2004
2. N. Jacobson, Basic Algebra, Dover, 1985.

PGMT402: Functional Analysis

Learning Outcomes:

1. Explain the fundamental concepts of Functional analysis and their role in modern mathematics.
2. Utilize the concepts of functional analysis, for example continuous and bounded operators, normed spaces and Hilbert spaces.
3. Study the behavior of different mathematical expressions arising in science and engineering.
4. Define normed linear space, Banach space, quotient space of a normed linear space.
5. Understand Riesz lemma and its application to infinite dimensional normed linear space.
6. Define bounded linear transformation and give its equivalent characterizations.
7. Understand and apply the Hahn-Banach theorem.
8. Understand and apply fundamental theorems from the theory of normed and Banach spaces including the Hahn-Banach theorem, the open mapping theorem, the closed graph theorem and the uniform boundedness theorem.
9. Explain the concept of projection on Hilbert and Banach spaces.

Unit I Hilbert spaces (15 Lectures)

Hilbert spaces, examples of Hilbert spaces such as $l^2, L^2(-\pi, \pi)\ and\ L^2(R^n)$. Bessel's inequality. Complete orthonormal set and maximal orthonormal basis. Orthogonal decomposition. Separable Hilbert space, Existence of a maximal orthonormal basis. Parseval's identity. Baire category theorem and applications.

Unit II. Normed Linear Spaces (15 Lectures)

Normed Linear spaces. Banach spaces and examples. Quotient space of a normed linear space. $l^p(1 \leq p \leq \infty)$ spaces are Banach spaces.
$L^p(\mu)(1 \leq p \leq \infty)$ spaces: Holder's inequality, Minkowski's inequality, $L^p(\mu)(1 \leq p \leq \infty)$ are Banach spaces.Finite dimensional normed linear spaces, Equivalent norms, Riesz Lemma and application to normed linear spaces

Unit III. Bounded Linear Transformations (15 Lectures)

Bounded linear transformations, Equivalent characterizations. The space $B(X, Y)$ Completeness of $B(X, Y)$ when Y is complete, dual space of a normed linear space.Riesz Representation theorem for Hilbert spaces. Hahn-Banach theorem.

Unit IV. Basic Theorems (15 Lectures)

Applications of Hahn-Banach theorem. Open mapping theorem, Closed graph theorem, Uniform boundedness Principle and their Applications.

Recommended Text Books:

1. G. F. Simmons, Introduction to Topology and Modern Analysis, Tata MacGrahill.
2. B.V. Limaye, Functional Analysis, Wiley Eastern.
3. Royden, Real Analysis, Macmillian.
4. E. Kreyszig, Introductory Functional Analysis with Applications, Wiley India.

5. J.R. Munkres, Topology, Pearson.

PGMT403:Partial Differential Equations

Learning Outcomes:

1. Learn how partial differential equations are used to study various physical problems.
2. Classify partial differential equations and transform into canonical form.
3. Solve linear and non-linear partial differential equation of first order by using Lagrange's and Charpit's method.
4. Determine the solutions of linear PDE's of second order with constant coefficients.
5. Classify second order PDE and solve standard PDE using separation of variable method.
6. Explain the properties of the Gaussian kernel.
7. Find the solution of the initial value problems of heat operator.
8. Formulate and solve significant PDE's like wave equation.
9. Explain the wave operators in one dimensional, two dimensional and three dimensional.

Unit I. First Order Partial Differential Equation (15 Lectures)

First order quasi-linear PDE in two variables: Integral surfaces, Characteristic curves, Cauchy's method of characteristics for solving First order quasi-linear PDE in two variables.
First order non-linear PDE in two variables, Characteristic equations, Characteristic strip, Cauchy problem and its solution for first order non linear PDE in two variables.

Unit II. Laplace operator (15 Lectures)

Symmetry properties of the Laplacian, basic properties of the Harmonic functions, the Fundamental solution, the Dirichlet's and Neumann boundary value problems, Green's function. Applications to the Dirichlet's problem in a ball in $R^n$and in a half space of R^n. Maximum Principle for bounded domains in R^n and uniqueness theorem for the Dirichlet boundary value problem.

Unit III. Heat operator (15 Lectures)

The properties of the Gaussian kernel, solution of initial value problem
$u_t - \Delta u$=0 for x$\in R^n$& t > 0 and u(x, 0) = f(x), (x $\in R^n$). Maximum principle for the heat equation and applications.

Unit IV. Wave operator (15 Lectures)

Wave operator in dimensions 1, 2 & 3; Cauchy problem for the wave equation. D Alemberts solution, Poisson formula of spherical means, Hadamard's method of descent, Inhomogeneous Wave equation.

Recommended Text Books:

1. F. John, Partial Differential Equations, Narosa publications.
2. G.B. Folland, Introduction to partial differential equations, Prentice Hall.
3. An elementary course in Partial Differential equations, by T. Amarnath.
4. Partial Differential equations by Phoolan Prasad and Renuka Ravindran.

PGMT404&PGMT405: ELECTIVE COURSES

Elective Courses to be selected from the following four courses.

A. NUMERICAL METHODS II

All methods discussed in the course should be illustrated using suitable programming language and conducting practicals.

Learning Outcomes:

1. Understand the theoretical and practical aspects of the use of numerical analysis.
2. Implement numerical methods for a variety of multidisciplinary applications.
3. Establish the advantages, disadvantages and limitations of numerical analysis.
4. Evaluate numerical integration using trapezoidal rule, Simpson's rule etc.
5. Approximate functions using least square approximations, weighted least square method etc.
6. Explain Gram-Schmidt orthogonalisation process.
7. Find the solutions of differential equations with constant coefficients using various methods such as Euler's modified method, Runge-Kutta method, etc.
8. Discuss the stability of numerical methods.
9. Classify partial differential equations.
10. Find the numerical solution of partial differential using appropriate methods.

Unit I. Numerical Integration (15 Lectures)
Numerical Integration: Newton-Cotes quadrature formula, Trapezoidal rule, Simpson's one third and three eighth rules, Errors in trapezoidal and Simpson's rules, Romberg's method, Gaussian quadrature, Multiple integrals.

Unit II. Approximation of functions (15 Lectures)
Least squares approximation, Weighted least squares method, Gram-Schmidt orthogonalizing process, Least squares approximation by Chebyshev polynomials, Discrete Fourier transforms, Fast Fourier Transforms.

Unit III. Differential Equations (15 Lectures)
Differential equations: Solutions of linear differential equations with constant coefficients, Series solutions, Euler's modified method, Runge-Kutta methods, Predictor corrector Methods, Stability of numerical methods.

Unit IV. Numerical Solutions of partial differential Equations (15 Lectures)
Classification, Finite difference approximations to derivatives, Numerical methods of solving elliptic, Parabolic and hyperbolic equations.

Reference Books:

(1) H.M.Antia , Numerical Analysis for scientists and engineers, TMH 1991.
(2) Jain, Iyengar, Numerical methods for scientific and engineering problems, New Age International, 2007.
(3) S.S.Sastry, Introductory methods of numerical analysis, Prentice-Hall India, 1977.
(4) K.E. Atkinson, An introduction to numerical analysis, John Wiley and sons, 1978.

B. Graph Theory-II

Learning Outcomes:

1. Solve problems involving edge and vertex coloring.
2. Calculate chromatic number and find chromatic polynomial.
3. Explain the properties of chromatic polynomial of a graph.
4. Define a planar and outer planar graph.
5. Derive Euler's formula.
6. Establish the non-planarity of $K_5, K_{3,3}$.
7. Define the directed graphs, directed cycles and explain the concept of networks.
8. Understand the maximum flow minimum cut theorem and Ford Fulkerson algorithm.
9. Find the spectrum and characteristic polynomial of a graph.
10. Deduce that a connected graph with diameter d has d+1 Eigen values.

Unit I. Graph Coloring (15 Lectures)
Line Graphs, Edge coloring-edge chromatic number, Vizing theorem, Timetabling problem, Vertex

coloring- Vertex chromatic number, Critical graphs, Brook's theorem, Chromatic polynomial of a graph- $\pi_k(G) = \pi_k(G - e) - \pi_k(G.e)$ properties of chromatic polynomial of a graph, Existence of a triangle free graph with high vertex chromatic number, Mycieleski's construction.

Unit II. Planar Graph (15 Lectures)
Planar graph, Plane embedding of a graph, Stereographic projection, Dual of a plane graph, Euler formula, Non planarity of K_5 and $K_{3,3}$, Outer planar graph, Five color theorem, Sub-division, Kuratowski's theorem(Without Proof).

Unit III. Flow Theory (15 Lectures)
Directed graphs, Directed paths and directed cycle, Tournament, Networks, Max flow min cut theorem, Ford- Fulkerson Theorem and Algorithm.

Unit IV. Characteristic Polynomials (15 Lectures)
Spectrum of a graph, Characteristic polynomial of a graph, Coefficients of characteristic polynomial of a graph, Adjacency algebra A(G) of a graph G, Dimension of $A(G) \geq diam(G) + 1$, A connected graph with diameter d has at least d + 1 eigen values, Circulant matrix, Determination of spectrum of graphs.

Reference Books

(1) J. A.Bondy and U.S.R.Murty, Graph Theory with Applications, The Macmillan Press,1976.
(2) J. A. Bondy and U. S. R. Murty, Graph Theory GTM Springer, 2008.
(3) M. Behzad and G. Chartrand, Introduction to the Theory of Graphs, Allyn and Becon Inc.,Boston, 1971.
(4) K. Rosen, Discrete Mathematics and its Applications, Tata-McGraw Hill,2011.
(5) D.B.West, Introduction to Graph Theory, Prentice-Hall, India, 2009.
(6) N. Biggs, Algebraic Graph Theory, Prentice-Hall, India.

C. Fourier Analysis

Learning Outcomes:

1. Find Fourier series of a periodic function.
2. Derive Dirichlet's Kernel, Bessel's inequality for a 2π periodic function.
3. Explain the convergence theorem.
4. Define Lebesgue integral.
5. Understand the Riemann-Lebesgue lemma and establish that the converse of the theorem is not true.
6. Derive Fejer's kernel and explain the Fejer's theorem.
7. Explain the convergence of Fourier series of an L^2-periodic function.
8. Explain the Dirichlet's problem for a unit disc.
9. Derive the Poisson kernel.
10. Find solution of Dirichlet's problem for the disc.

Unit I. Fourier series (15 Lectures)
The Fourier series of a periodic function, Dirichlet kernel, Bessel's inequality for a 2π-periodic Riemann integrable function, convergence theorem for the Fourier series of a 2π-periodic and piecewise C^1-function, uniqueness theorem (If f, g are 2π-periodic and piecewise smooth function having same Fourier coefficients, then f = g).
Relating Fourier coefficients of f and f' where f is continuous 2π-periodic and piecewise C^1-function and a convergence theorem: If f is continuous 2π-periodic and piecewise C^1-function, then the Fourier series of f converges to f absolutely and uniformly on R.

Unit II. Dirichlet's theorem (15 Lectures)
Review: Lebesgue measure of R, Lebesgue integrable functions, Dominated Convergence theorem, Bounded linear maps (no questions be asked).
Definition of Lebesgue integrable periodic functions (i.e. L^1-periodic), Fourier Coefficients of L^1-periodic functions, L^2-periodic functions. Any L^2-periodic function is L^1-periodic. Riemann-Lebesgue Lemma. Converse of Riemann-Lebesgue lemma does not hold (ref: W. Rudin, Real and Complex Analysis, Tata McGraw Hill).
Bessel's inequality for a L^2-periodic function. Dirichlet's Theorem on point-wise convergence of Fourier series (If f is Lebesgue integrable periodic function that is differentiable at a point x_0; then the Fourier series of f at x_0 converges to $f(x_0)$) and convergence of the Fourier series of functions such as $f(x) = |x|$ on $[-\pi, \pi]$.

Unit III. Fejer's Theorem and applications (15 Lectures)

Fejer's Kernel, Fejer's Theorem for a continuous 2π-periodic, density of trigonometric polynomials in $L^2(-\pi, \pi)$, Parseval's identity.
Convergence of Fourier series of an L^2-periodic function w.r.t the L^2-norm, Riesz-Fischer theorem on Unitary isomorphism from $L^2((-\pi, \pi)$ onto the sequence space l^2 of square summable complex sequences.

Unit IV. Dirichlet Problem in the unit disc (15 Lectures)
Laplacian, Harmonic functions, Dirichlet Problem for the unit disc, The Poisson kernel, Abel summability, Abel summability of periodic continuous functions, Weierstrass Approximation Theorem as application, Solution of Dirichlet problem for the disc.
Applications of Fourier series to Isoperimetric inequality in the plane and Heat equation on the circle.

D. Calculus on Manifolds

Learning Outcomes:
1. Learn the basic concepts of differentiable manifolds and tensor calculus.
2. Acquire essential knowledge of tensor calculus for further studies in Riemannian geometry, and related areas in mathematics.
3. Define a multilinear map on a finite dimensional vector space over real numbers.
4. Explain wedge products and its properties.
5. Define sub manifolds of R^n, smooth functions defined on submanifolds, orientable submanifolds, vector field on submanifolds.
6. Find tangent vectors and tangent spaces of sub manifolds.
7. Appreciate the unification of Green's theorem, Stoke's theorem and Divergence theorem.

Unit I. Multilinear Algebra (15 Lectures)
Multilinear map on a finite dimensional vector space V over R; and k-tensors on V, the collection $T^k(V)$ (or $\otimes^k(V^*)$) of all k-tensors on V, tensor product $S\otimes T$ of $S \in T^k(V)$ & $T \in T^k(V)$, Alternating tensors and the collection $\wedge^k V^*$ of all k-tensors on V; The exterior product (or wedge product, basis of $\wedge^k V^*$; orientations of a finite dimensional vector space V over R.

Unit II. Differential Forms (15 Lectures)
Differential forms: k-forms on R^n, wedge product $\omega \wedge \eta$ of a k-form ω and l-form η ; the exterior derivative and properties, Pull back of forms and properties, Closed and exact forms, Poincare's lemma.

Unit III. Basics of submanifolds of R^n (15 Lectures)

Submanifolds of R^n; submanifolds of R^n with boundary, Smooth functions defined on Submanifolds of R^n, Tangent vectors and Tangent spaces of Submanifolds of R^n.
p-forms and differentiable p-forms on a submanifold of R^n; exterior derivative $d\omega$ of any differentiable p-form on a submanifolds of R^n, Orientable submanifolds of R^n and Oriented submanifolds of R^n, Orientation preserving maps, Vector fields on submanifolds of R^n, outward unit normal on the boundary of a submanifold of R^n with non-empty boundary, induced orientation of the boundary of an oriented submanifold of R^n with non-empty boundary.

Unit IV: Stoke's Theorem (15 Lectures)

Integral $\int_{[0,1]^k} w$ of a k-form on the cube $[0,1]^k$, integral $\int_c w$ of a k-form on an open subset A of R^k where c is a singular k-cube in A. Theorm(Stokes's Theorem for k-cubes): If w is a (k-1)-form on an open subset A of R^k and c is a singular k-cube in A then $\int_c dw = \int_{\partial c} w$
Integration of a differentiable k-form on an oriented k-dimensional submanifold M of R^n : Change of variables theorem: If $c_1, c_2 : [0, 1]^k \to M$ are two Orientation preserving maps in M and w is any k-form on M such that w = 0 outside of $c_1([0, 1]^k) \cap c_2([0, 1]^k)$, then $\int_{c_1} w = \int_{c_2} w$, Stokes' theorem for submanifolds of R^k, Volume element, Integration of functions on a submanifold of R^k, Classical theorems: Green's theorem, Divergence theorem of Gauss, Green's identities.

Reference Books:

1. V. Guillemin and A. Pollack, Differential Topology, AMS Chelsea Publishing, 2010.
2. J. Munkres, Analysis on Manifolds, Addision Wesley.
3. A. Browder, Mathematical Analysis, Springer International edition.

PGMT406
PROJECT COURSE

Learning Outcomes:

1. Students will acquire the ability to make links across different areas of knowledge and to generate, develop and evaluate ideas and information so as to apply these skills to the project task.
2. Students will acquire the skills to communicate effectively and to present ideas clearly and coherently to specific audience in both the written and oral forms.
3. Students will acquire collaborative skills through working in a team to achieve common goals.
4. Students will be able to learn on their own, reflect on their learning and take appropriate actions to improve it.

Scheme of Examination

I: The scheme of examination for the syllabus of Semesters III& IV of M.Sc. Programme (CBCS) in the subject of Mathematics will be as follows.

A. There shall be a **Semester-end examination with 60 marks**.

Duration: - Examination shall be of **2.5 Hours** duration.

Theory Question Paper Pattern:-

1) There shall be **five** questions each of **12 marks**.

2) All questions shall be compulsory with internal choice within each question.

3) Each question may be subdivided into sub-questions a, b, c, and the allocation of

marks dependson the weightage of the topic.

4) Each question will be of 18 marks when marks of all the sub-questions are added

(including theoptions) in that question.

Questions		Marks
Q1	Based on Unit I	12
Q2	Based on Unit II	12
Q3	Based on Unit III	12
Q4	Based on Unit IV	12
Q5	Based on Units I,II,III& IV	12
	Total Marks	60

B. There shall be **Continuous internal Assessment for 40 marks** of each paper of which unit test of 20 marks will be conducted and the other 20 marks will be awarded for project-power point presentation.

II:Evaluation of Project work:

The evaluation of the Project submitted by a student shall be made by a Committee appointed by the Head of the Department of Mathematics of the college.

The presentation of the project is to be made by the student in front of the committee appointed by the Head of the Department of Mathematics. This committee shall have two members, possibly with one external referee.

The Marks for the project are detailed below:

1. Contents of the project: 40 marks.
2. Presentation of the project: 30 marks.
3. Viva of the project: 30 marks.
4. Total Marks = 100 per project per student.